ANALYSIS OF CONTINUOUS RIGID FRAMES WITH JOINT TRANSLATION PREVENTED BY CARRY-OVER SLOPES AND CARRY-OVER MOMENTS

By

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ANALYSIS OF CONTINUOUS RIGID FRAMES

WITH JOINT TRANSLATION PREVENTED

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PREFACE

This study is the first in a series of studies to develop analyses of continuous frames by carry-over methods under the direction of Professor Jan J. Tuma. This presentation consists of the analysis of frames without joint translation.

I wish to express my indebtedness and gratitude to Professor Tuma for his invaluable aid and constructive criticism in the preparation of this thesis, and for his excellent instruction in principles of structural analysis. Acknowledgement is also due all other faculty and student members of the School of Civil Engineering who have contributed to my education and made my stay here a pleasant one.

I am indebted to my wife, Betty Jo, for making my graduate study possible and for her careful typing of the manuscript.

R.G.G.

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NOMENCLATURE

Θ_j	Slope at j
Θ_j^*	Starting slope at j
Σ	Summation sign
C_{ij} C_{ji}	Slope deflection carry-over factors for member ij
D_{ij}	Distribution factor at i of member ij
E	Modulus of elasticity
FM_{ij}	Fixed-end moment at i on member ij due to loads
I	Moment of inertia
JM_j	Joint moment at j
K_{ij}	Stiffness at i of member ij with j fixed
K''_{ij}	Stiffness at i of member ij with $\Theta_j = -\Theta_i$
$\bar{K}_{ij}$	Relative stiffness at i of member ij
L_{ij}	Length of member ij
M_{ij}	Bending moment at i on member ij
m_j	Starting joint moment at j
r_{ij} r_{ji}	Joint moment carry-over factors for member ij
t_{ij} t_{ji}	Slope carry-over factors for member ij
w	Uniform load per unit length of span

PART I

INTRODUCTION

A method for the analysis of coplanar continuous rigid frames with straight members and with joint translation prevented is presented. Any type of coplanar loading may be considered acting on the structure.

Moments of inertia of individual members may be constant or variable. The customary assumption is made that deformation due to shear is small in comparison with that due to moment and is neglected.

From the general slope deflection equations for a straight elastic member, an equilibrium equation is obtained for each rigid joint of the frame. Two procedures of analysis, the carry-over slope method and the carry-over moment method, are derived from the system of joint equations obtained.

The derivation of the carry-over slope equation was presented by Professor Jan J. Tuma in June, 1958 for the analysis of continuous trusses. The writer's contribution is the extension of this equation to continuous frame analysis. The derivation of the joint moment carry-over procedure shown in Part III was presented by Professor Tuma in the Fall of 1958. The writer's contribution here is the numerical application of this approach and the explanation of its correlation with the carry-over slope method.

The application of both methods is similar to that of the moment distribution method except for the distribution of fixed-end moments (1). A different form of numerical successive approximation is used to evaluate the final end moments. Physically the process can be described as follows:

(a) With all joints assumed fixed, fixed-end moments are calculated.

(b) Then each joint is allowed to rotate independently and corresponding starting slopes and starting joint moments, terms to be defined later, are computed, one for each joint.

(c) By calculating the effect upon each joint of all its adjacent joints to any desired degree of accuracy through a direct carry-over process the final configuration of the frame is obtained, from which the end moments are easily calculated.

Inherent characteristics of the carry-over methods in frames are quite analogous to those of the carry-over moment method in beams (2) in that:

(a) One starting slope or moment is computed at each joint.

(b) One final slope or moment is obtained at each joint.

(c) No distribution of unbalances is required during the relaxation procedure.

(d) The relaxation procedure is accomplished by means of carry-over factors only.

(e) The procedure is self-checking.

Both methods are applied to two numerical examples (Parts IV and V).

First a simple continuous building frame is analyzed to demonstrate the procedure. Then a symmetrical multi-story building frame is analyzed for a symmetrical loading to show the application of the methods to more complicated structures. Both problems are also solved by the moment distribution method as a check on the computations.

PART II

CARRY-OVER SLOPE METHOD

1. Joint Equation

A portion of a continuous rigid frame acted upon by a general system of loads is removed for investigation (Fig. 1).

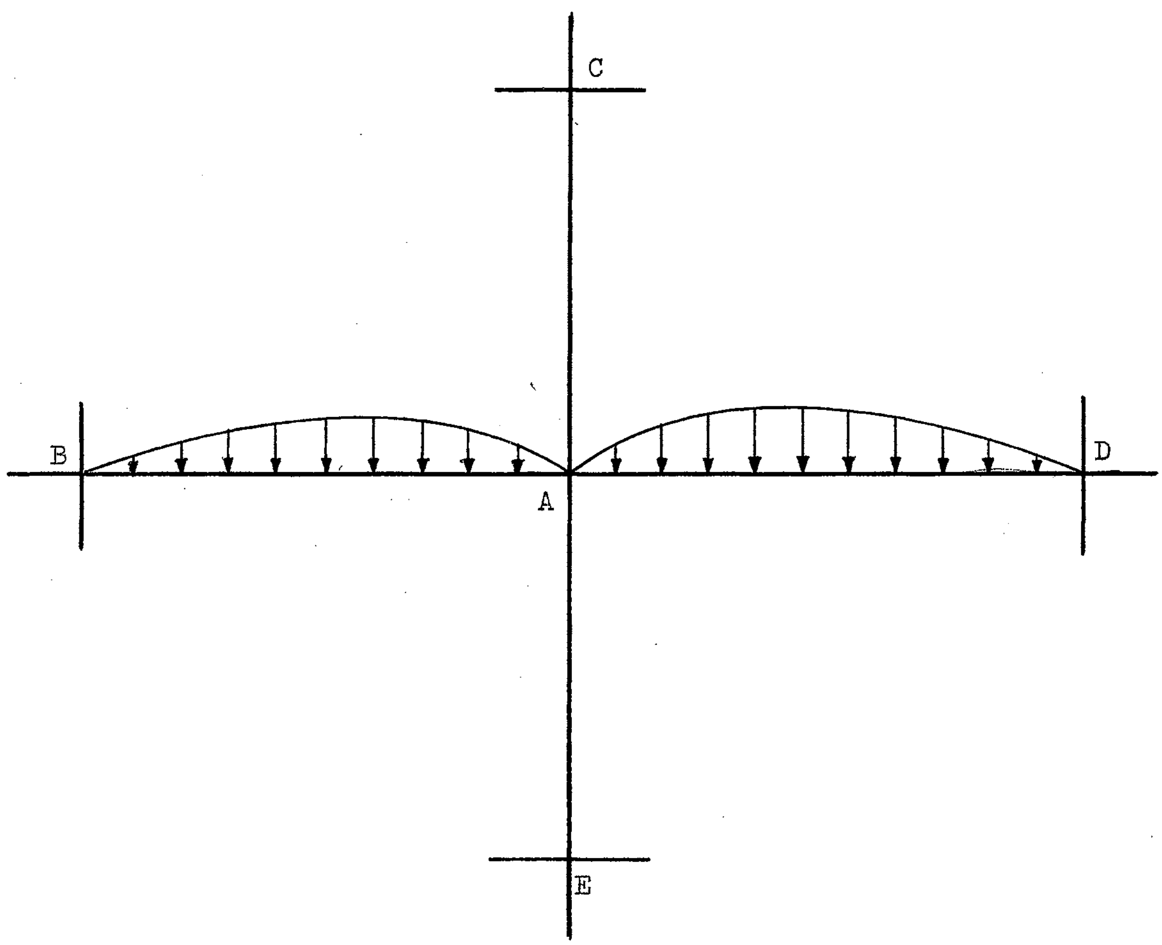


Figure 1

Portion of a Continuous Frame

The slope-deflection equations for the members framing into joint A are:

$$
\begin{aligned}
M_{AB} &= K_{AB}\,\theta_A + C_{BA}\,K_{BA}\,\theta_B + FM_{AB} \\
M_{BA} &= K_{BA}\,\theta_B + C_{AB}\,K_{AB}\,\theta_A + FM_{BA} \\
M_{AC} &= K_{AC}\,\theta_A + C_{CA}\,K_{CA}\,\theta_C + FM_{AC} \\
M_{CA} &= K_{CA}\,\theta_C + C_{AC}\,K_{AC}\,\theta_A + FM_{CA} \\
M_{AD} &= K_{AD}\,\theta_A + C_{DA}\,K_{DA}\,\theta_D + FM_{AD} \\
M_{DA} &= K_{DA}\,\theta_D + C_{AD}\,K_{AD}\,\theta_A + FM_{DA} \\
M_{AE} &= K_{AE}\,\theta_A + C_{EA}\,K_{EA}\,\theta_E + FM_{AE} \\
M_{EA} &= K_{EA}\,\theta_E + C_{AE}\,K_{AE}\,\theta_A + FM_{EA}
\end{aligned} \tag{1}
$$

In equations (1) clockwise rotations are considered to be positive.

Provided that no external couple is applied at A in addition to the loading shown we must have the condition of joint equilibrium:

$$\Sigma M_A = M_{AB} + M_{AC} + M_{AD} + M_{AE} = 0 \tag{2}$$

Substituting corresponding equalities into equation (2) we get:

$$0 = \theta_A \Sigma K_A + C_{BA}K_{BA}\theta_B + C_{CA}K_{CA}\theta_C + C_{DA}K_{DA}\theta_D + C_{EA}K_{EA}\theta_E + \Sigma FM_A \tag{3}$$

Solving for θ_A:

$$\theta_A = -\frac{C_{BA}K_{BA}}{\Sigma K_A}\,\theta_B - \frac{C_{CA}K_{CA}}{\Sigma K_A}\,\theta_C - \frac{C_{DA}K_{DA}}{\Sigma K_A}\,\theta_D - \frac{C_{EA}K_{EA}}{\Sigma K_A}\,\theta_E - \frac{\Sigma FM_A}{\Sigma K_A} \tag{4}$$

Equation (4) is seen to be a five-slope equation, a term found useful for its description because of its similarity with the well-known three-moment equation used in continuous beam analysis.

2. Carry-over Slope Equation

Equation (4) written for joint j (any joint) is:

$$\theta_j = -\sum \frac{C_{ij}K_{ij}}{\Sigma K_j}\;\theta_i - \frac{\Sigma FM_j}{\Sigma K_j} \text{ where i is any joint adjacent to j.} \tag{5}$$

Denoting $$-\frac{C_{ij}K_{ij}}{\Sigma K_j} = t_{ij} \tag{6}$$

and $$-\frac{\Sigma FM_j}{\Sigma K_j} = \theta_j^* \tag{7}$$

the final form of equation (5) becomes:

$$\theta_j = \theta_j^* + \Sigma t_{ij}\theta_i \tag{8}$$

Writing equation (8) for each joint of the frame yields a system of n equations in n unknowns. Each equation consists of the three basic terms; starting slope θ_j^*, carry-over factors t_{ij}, and redundant slopes.

Solution of the system of equations by the carry-over procedure begins using the initial approximations:

$$\theta_j \doteq \theta_j^*$$

Successive corrections to these approximate values of θ_j are computed through the use of the carry-over factors t_{ij}. The direct and modified procedures available for computation of these corrections are demonstrated in Part IV.

3. Starting Slope

The starting slope θ_j^* is the rotation of joint j due to loads if all adjacent joints i are fixed (Fig. 2).

Since $\theta_i = 0$, equation (8) gives:

$$\theta_j = \theta_j^* = -\frac{\Sigma FM_j}{\Sigma K_j}$$

4. Carry-over Slope Factor

The carry-over factor t_{ij} is the rotation produced at joint j by a unit rotation of joint i if all other joints adjacent to j are fixed (Fig. 3).

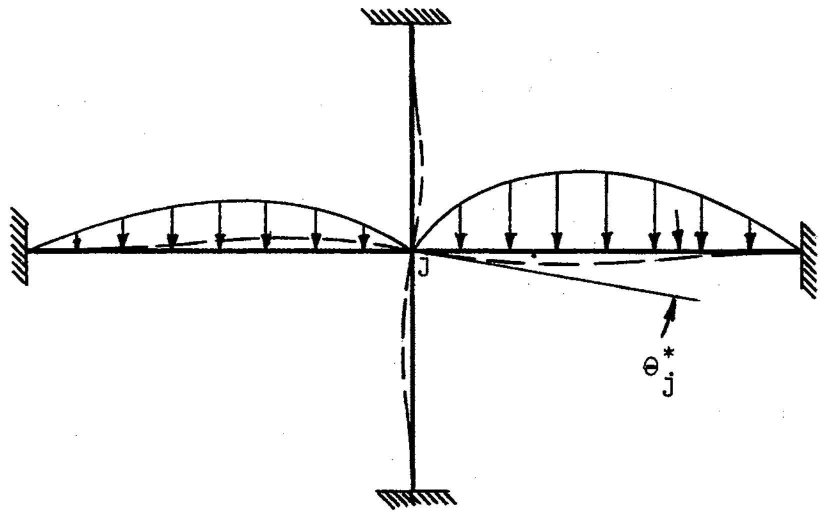


Figure 2

Physical Interpretation of Starting Slope

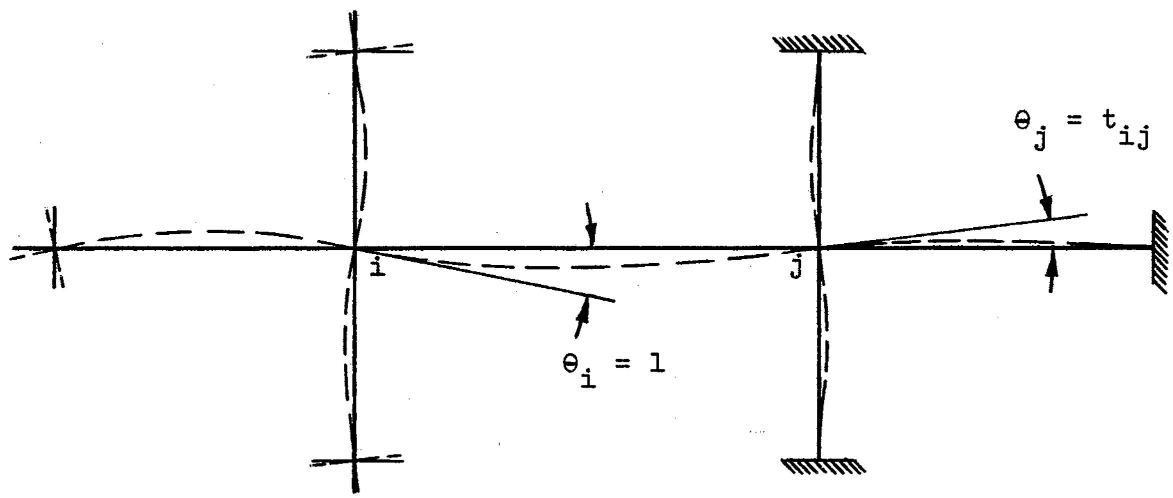


Figure 3

Physical Interpretation of Carry-over Slope Factor

With the effect of loads on the rotation of j illustrated in Figure 2, the additional effect of rotation of any joint adjacent to j is illustrated in Figure 3. Since for this case $\theta_j^* = 0$ we have from equation (8):

$$\theta_j = t_{ij}\theta_i$$

For $\theta_i = 1$:

$$\theta_j = t_{ij} = -\frac{C_{ij}K_{ij}}{\Sigma K_j}$$

PART III

CARRY-OVER MOMENT METHOD

1. Carry-over Moment Equation

A new term, joint moment, designated by JM_j, is introduced and defined by the equation:

$$JM_j = \theta_j \Sigma K_j \tag{9}$$

Substitution into equation (5) yields:

$$JM_j = -\sum \frac{C_{ij}K_{ij}}{\Sigma K_i} JM_i - \Sigma FM_j \tag{10}$$

$$JM_j = - \Sigma C_{ij}D_{ij}JM_i - \Sigma FM_j \tag{11}$$

Denoting $\qquad - C_{ij}D_{ij} = r_{ij} \qquad (12)$

and $\qquad - \Sigma FM_j = m_j \qquad (13)$

the final form of equation (11) becomes:

$$JM_j = m_j + \Sigma r_{ij}JM_i \tag{14}$$

Equation (14) written for joint A of the frame in Fig. 1 is:

$$JM_A = m_A + r_{BA}JM_B + r_{CA}JM_C + r_{DA}JM_D + r_{EA}JM_E \tag{15}$$

Equation (15) is conveniently referred to as the general five-moment equation.

Writing equation (14) for each joint of the frame again yields a system of n equations in n unknowns. Each equation consists of the three basic terms; starting moment m_j, carry-over factors r_{ij}, and redundant joint moments. Solution of the system of equations by the carry-over procedure begins using the initial approximations:

$$JM_j \doteq m_j$$

Successive corrections to these approximate values of JM_j are computed through the use of the carry-over factors r_{ij}.

2. Starting Moment

The starting moment m_j is the joint moment developed at j by the loads if all adjacent joints i are fixed (Fig. 4).

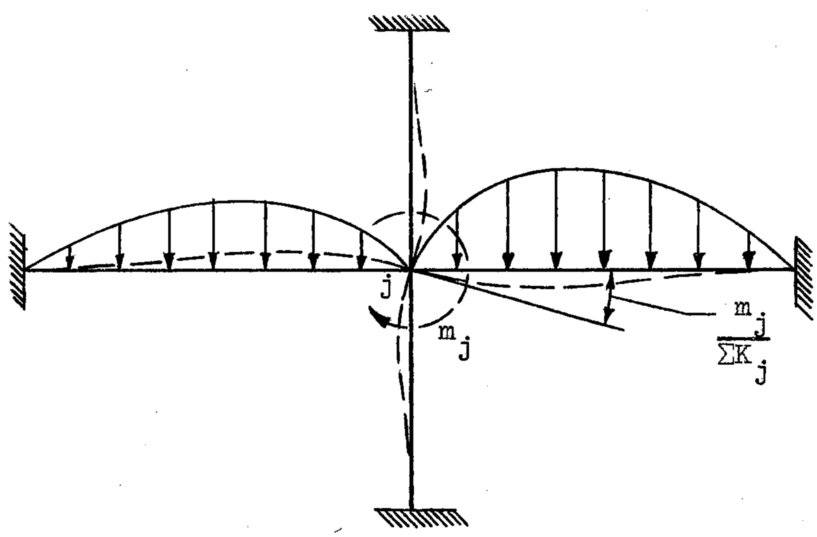


Figure 4

Physical Interpretation of Starting Moment

Since $\theta_i = 0$ then $JM_i = 0$ from equation (9) and equation (14) gives:

$$JM_j = m_j = - \Sigma FM_j$$

3. Carry-over Moment Factor

The carry-over factor r_{ij} is the joint moment developed at j from a unit joint moment at i, if all other joints adjacent to j are fixed (Fig. 5).

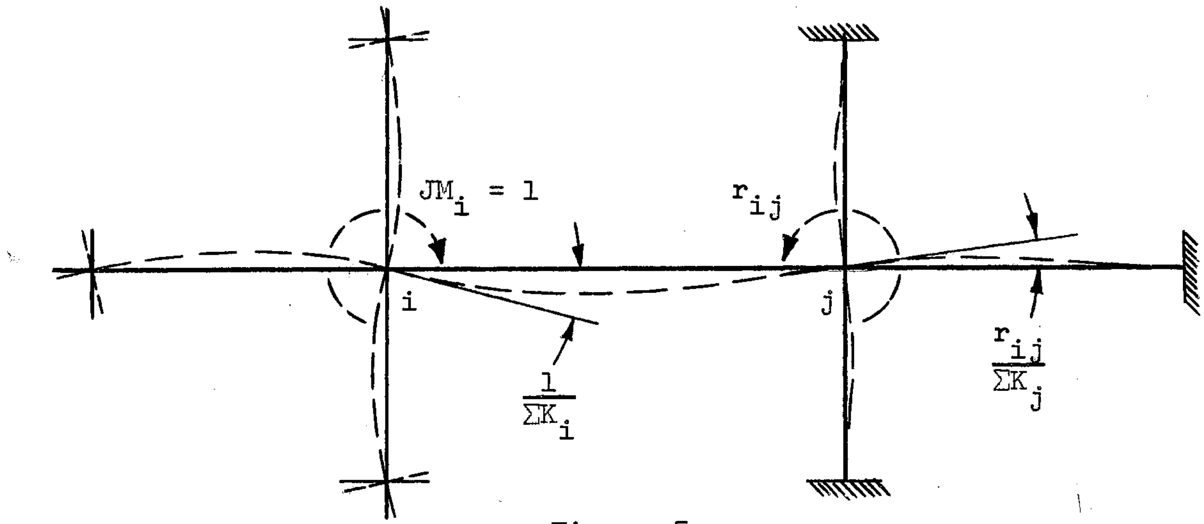


Figure 5

Physical Interpretation of Carry-over Moment Factor

With the effect of loads on JM_j illustrated in Figure 4 the additional effect of the joint moment at any joint adjacent to j is illustrated in Figure 5. Since for this case $m_j = 0$ we have from equation (14):

$$JM_j = r_{ij} JM_i$$

For $JM_i = 1$:

$$JM_j = r_{ij} = - C_{ij} D_{ij}$$

4. End Moments

Expressions for the end moments in any member ij in terms of the joint moments are obtained by substitution of equation (9) into the slope deflection equations for member ij:

$$\begin{aligned} M_{ij} &= K_{ij}\theta_i + C_{ji}K_{ji}\theta_j + FM_{ij} \\ M_{ji} &= K_{ji}\theta_j + C_{ij}K_{ij}\theta_i + FM_{ji} \end{aligned} \tag{16}$$

$$\begin{aligned} M_{ij} &= \frac{K_{ij}JM_i}{\Sigma K_i} + \frac{C_{ji}K_{ji}JM_j}{\Sigma K_j} + FM_{ij} \\ M_{ji} &= \frac{K_{ji}JM_j}{\Sigma K_j} + \frac{C_{ij}K_{ij}JM_i}{\Sigma K_i} + FM_{ji} \end{aligned} \tag{17}$$

$$M_{ij} = D_{ij}JM_i + C_{ji}D_{ji}JM_j + FM_{ij}$$
$$M_{ji} = D_{ji}JM_j + C_{ij}D_{ij}JM_i + FM_{ji} \tag{18}$$

$$M_{ij} = D_{ij}JM_i - r_{ji}JM_j + FM_{ij}$$
$$M_{ji} = D_{ji}JM_j - r_{ij}JM_i + FM_{ji} \tag{19}$$

As will be shown in Part IV, the calculation of end moments can be conveniently combined with the checking procedure in tabular form if desired.

PART IV

EXAMPLE PROBLEM 1

The carry-over methods are applied to a typical building frame analysis problem (Fig. 6). The haunched beam coefficients are taken from reference (3). This problem is based on conditions of design found in reference (4).

Relative stiffnesses shown in all solutions are obtained from the absolute stiffnesses computed as follows:

$$K_{ji} = \frac{4EI}{12} = .33EI \text{ for all columns}$$

$$K_{ji} = K_{ij} = \frac{10.85EI}{20} = .54EI \text{ for 20' beams}$$

$$K_{JK} = K_{KJ} = \frac{7.81EI}{30} = .26EI$$

where I is the moment of inertia of columns and center of beams. Sums of stiffnesses are shown encircled on sketches.

1. Carry-over Slope Solution (Table 1)

(a) Carry-over Slope Factors

$$C_{ij}K_{ij} = C_{ji}K_{ji} = \frac{7.65EI}{20} = .38EI \text{ for 20' beams}$$

$$C_{JK}K_{KJ} = C_{KJ}K_{KJ} = \frac{5.15EI}{30} = .17EI$$

$$t_{GH} = -\frac{.38}{1.74} = -.22 = t_{ML} = t_{JH} = t_{KL}$$

$$t_{HG} = -\frac{.38}{1.20} = -.32 = t_{LM}$$

$$t_{HJ} = -\frac{.38}{1.46} = -.26 = t_{LK}$$

$$t_{JK} = -\frac{.17}{1.46} = -.12 = t_{KJ}$$

(b) Starting Slopes

$$\theta^*_G = 0 \qquad \theta^*_H = -\frac{-0.1034(1)(20)^2}{1.74} = +\frac{23.8}{EI} = -\theta^*_L$$

$$\theta^*_J = -\frac{+.1034(1)(20)^2}{1.46EI} = \frac{-28.3}{EI} = -\theta^*_K$$

$$\theta^*_M = -\frac{-.5(1)(6)^2}{1.20EI} = +\frac{15.0}{EI}$$

(c) Carry-over Procedure (Tables 1 and 2)

The alternate carry-over procedure (2) is used in Table 1. The direct procedure is shown in Table 2 for comparison.

(d) Checking Procedure

The check is established if $\Sigma M_{ji} \doteq 0$.

2. Carry-over Moment Solution (Table 3)

(a) Carry-over Moment Factors

$$r_{GH} = -\frac{.38}{1.20} = -.32 = r_{ML}$$

$$r_{HG} = -\frac{.38}{1.74} = -.22 = r_{LM} = r_{HJ} = r_{LK}$$

$$r_{JH} = -\frac{.38}{1.46} = -.26 = r_{KL} \qquad r_{KJ} = -\frac{.17}{1.46} = -.12 = r_{JK}$$

(b) Starting Moments

$$m_G = 0 \qquad m_H = -(-.1034)(1)(20)^2 = +41.4 \text{ k-ft} = m_K$$

$$m_J = m_L = -41.4 \text{ k-ft} \qquad m_M = -(-.5)(1)(6)^2 = +18.0 \text{ k-ft}$$

(c) Carry-over Procedure

The alternate numerical procedure (2), recommended for general use, is again used in Table 3.

(d) Checking Procedure

Distribution factors are computed by referring to the sketch. As before, the check is established if $\Sigma M_{ji} \doteq 0$

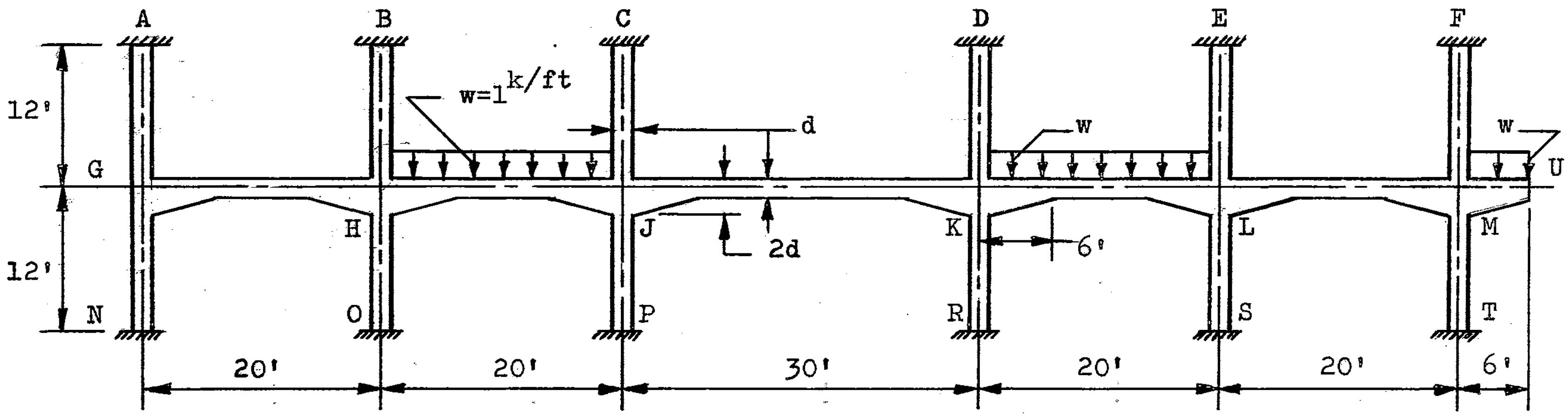
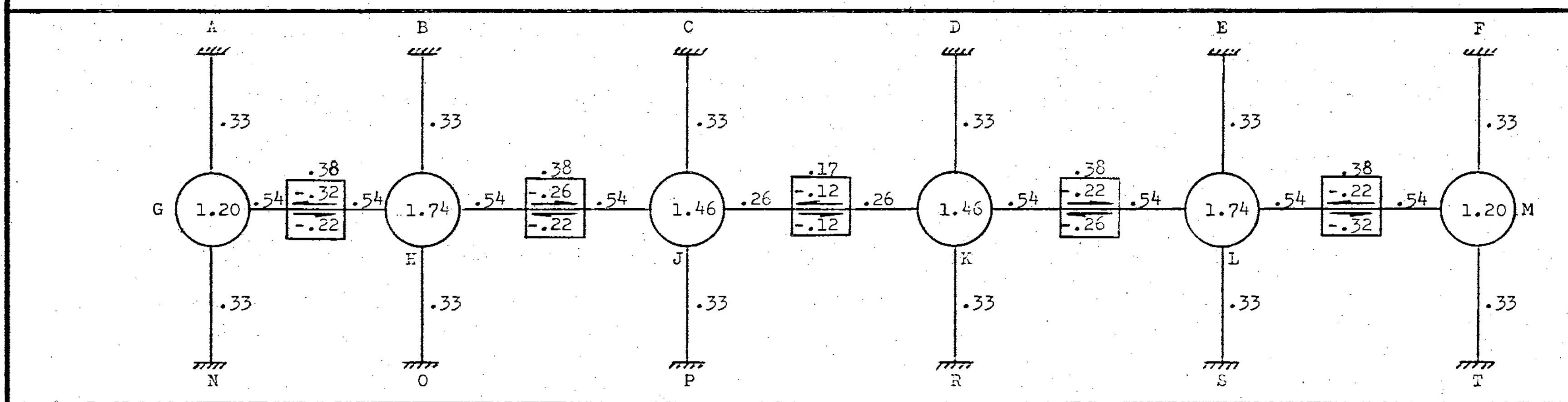


Figure 6

Continuous Building Frame

EXAMPLE 1 CARRY-OVER SLOPE SOLUTION TABLE 1



	G	H	J	K	L	M
e^*_j	0.0	+23.8	-28.3	+28.3	-25.8	+15.0
			- 6.2		- 6.2	
	- 7.6	+ 8.3	- 3.4	+ 4.5	- 3.3	
		+ 1.7	- 2.6	+ 8.7	- 2.9	+10.7
	- 3.2	+ 0.9	- 1.6	+ 0.5	- 2.4	
		+ 0.7	- 0.4	+ 1.4	- 0.4	+ 1.7
	- 0.5	+ 0.1	- 0.2	+ 0.1	- 0.4	
		+ 0.1		+ 0.2		+ 0.3
θ_j	-11.3	+35.6	-42.7	+43.7	-39.4	+27.7

ji	GA	GN	GH	HG	HB	HO	HJ	JH	JC	JP	JK	KJ	KO	KR	KL	LK	LE	LS	LM	ML	MF	MT
$K_{ji}e_j$	- 3.8	- 3.8	- 6.1	+19.2	+11.9	+11.9	+19.2	-23.1	-14.2	-14.2	-11.1	+11.4	+14.6	+14.6	+23.6	-21.3	-13.1	-13.1	-21.3	+15.0	+ 9.2	+ 9.2
$C_{ij}K_{ij}\theta_i$	0.0	0.0	+13.5	- 4.3	0.0	0.0	-16.2	+13.5	0.0	0.0	+ 7.4	- 7.3	0.0	0.0	-15.0	+16.6	0.0	0.0	+10.5	-15.0	0.0	0.0
FM_{ji}	0.0	0.0	0.0	0.0	0.0	0.0	-41.4	+41.4	0.0	0.0	0.0	0.0	0.0	0.0	-41.4	+41.4	0.0	0.0	0.0	0.0	0.0	0.0
M_{ji}	- 3.8	- 3.8	+ 7.4	+14.9	+11.9	+11.9	-38.4	+31.8	-14.2	-14.2	- 3.7	+ 4.1	+14.6	+14.6	-32.8	+36.7	-13.1	-13.1	-10.8	0.0	+ 9.2	+ 9.2

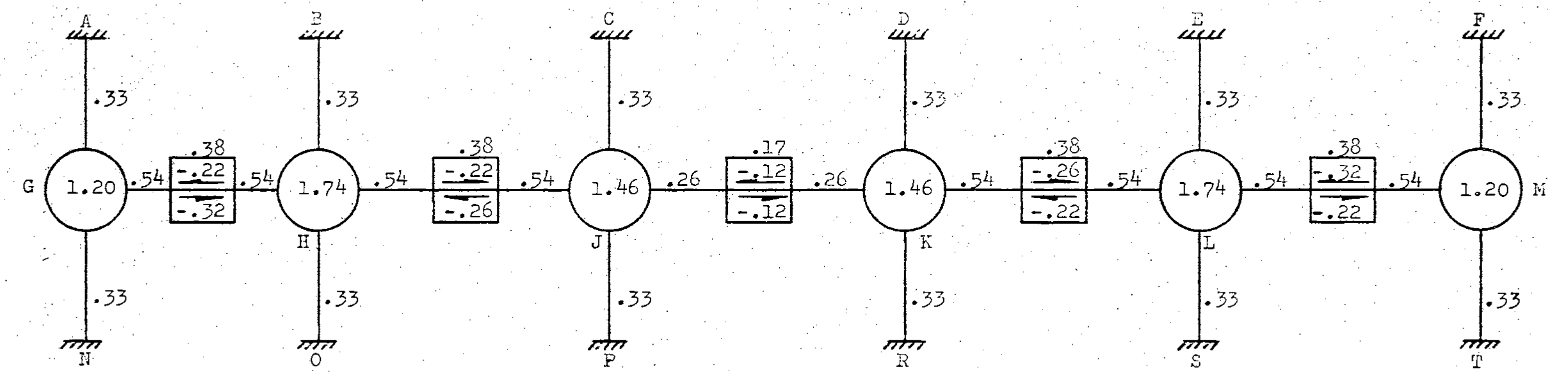
EXAMPLE 1 DIRECT CARRY-OVER SLOPE PROCEDURE TABLE 2

JOINT	G	H	J	K	L	M
CARRY-OVER FACTORS	→ -.22	← -.32 → -.26	← -.22 → -.12	← -.12 → -.22	← -.26 → -.32	← -.22
STARTING SLOPES	0	+23.8	-28.3	+28.3	-23.8	+15.0
1. C.O.S.	- 7.6	0	- 6.2	+ 3.4	- 6.2	+ 7.6
		+ 6.2	- 3.4	+ 6.2	- 3.3	
2. C.O.S.	- 2.0	+ 1.7	- 1.6	+ 1.2	- 2.1	+ 3.0
		+ 2.1	- 1.2	+ 2.5	- 1.7	
3. C.O.S.	- 1.2	+ 0.4	- 1.0	+ 0.3	- 0.8	+ 1.2
		+ 0.6	- 0.4	+ 1.0	- 0.7	
4. C.O.S.	- 0.3	+ 0.3	- 0.3	+ 0.2	- 0.3	+ 0.5
		+ 0.3	- 0.2	+ 0.4	- 0.3	
5. C.O.S.	- 0.2	+ 0.1	- 0.2	+ 0.1	- 0.1	+ 0.2
		+ 0.1	- 0.1	+ 0.1	- 0.1	
FINAL SLOPES	-11.3	+35.6	-42.9	+43.7	-39.4	+27.5

EXAMPLE 1

CARRY-OVER MOMENT SOLUTION

TABLE 3



	G	H	J	K	L	M
m_j	0.0	+41.4	-41.4	+41.4	-41.4	+18.0
			- 9.1		-10.8	
	- 9.1	+14.4	- 5.0	+ 6.7	- 5.8	
		+ 2.9	- 3.8	+12.8	- 5.1	+12.8
	- 3.8	+ 1.6	- 2.3	+ 0.7	- 4.1	
		+ 1.2	- 0.6	+ 2.0	- 0.7	+ 2.0
	- 0.6	+ 0.2	- 0.3	+ 0.1	- 0.6	
		+ 0.2		+ 0.3		+ 0.3
JM_j	-13.5	+61.9	-62.5	+64.0	-68.5	+33.1

ji	GA	GN	GH	HG	HB	HC	HJ	JH	JC	JP	JK	KJ	KD	KR	KL	LK	LE	LS	LM	ML	MF	MT
D_{ji}	.275	.275	.450	.310	.190	.190	.310	.370	.226	.226	.178	.178	.226	.226	.370	.310	.190	.190	.310	.450	.275	.275
$D_{ji}JM_j$	- 3.7	- 3.7	- 6.1	+19.2	+11.8	+11.8	+19.2	-23.1	-14.1	-14.1	-11.1	+11.4	+14.5	+14.5	+23.7	-21.2	-13.0	-13.0	-21.2	+14.9	+ 9.1	+ 9.1
$-r_{ij}JM_i$	0.0	0.0	+13.6	- 4.3	0.0	0.0	-16.2	+13.6	0.0	0.0	+ 7.7	- 7.5	0.0	0.0	-15.1	+16.6	0.0	0.0	+10.6	-15.1	0.0	0.0
FM_{ji}	0.0	0.0	0.0	0.0	0.0	0.0	-41.4	+41.4	0.0	0.0	0.0	0.0	0.0	0.0	-41.4	+41.4	0.0	0.0	0.0	0.0	0.0	0.0
M_{ji}	- 3.7	- 3.7	+ 7.5	+14.9	+11.8	+11.8	-33.4	+31.9	-14.1	-14.1	- 3.4	+ 3.9	+14.5	+14.5	-32.8	+36.8	-13.0	-13.0	-10.6	- 0.2	+ 9.1	+ 9.1

3. Moment Distribution Solution (Table 4)

(a) Carry-over Factors

$$C_{ji} = C_{ij} = \frac{0.38EI}{0.54EI} = 0.70 \text{ for 20' girders}$$

$$C_{JK} = C_{KJ} = \frac{0.17EI}{0.26EI} = 0.65$$

(b) Fixed-end Moments

$$FM_{HJ} = FM_{KL} = -.1034(1)(20)^2 = -41.4 \text{ k-ft}$$

$$FM_{JH} = FM_{LK} = +.1034(1)(20)^2 = +41.4 \text{ k-ft}$$

$$FM_{MU} = -.5(1)(6)^2 = -18.0 \text{ k-ft}$$

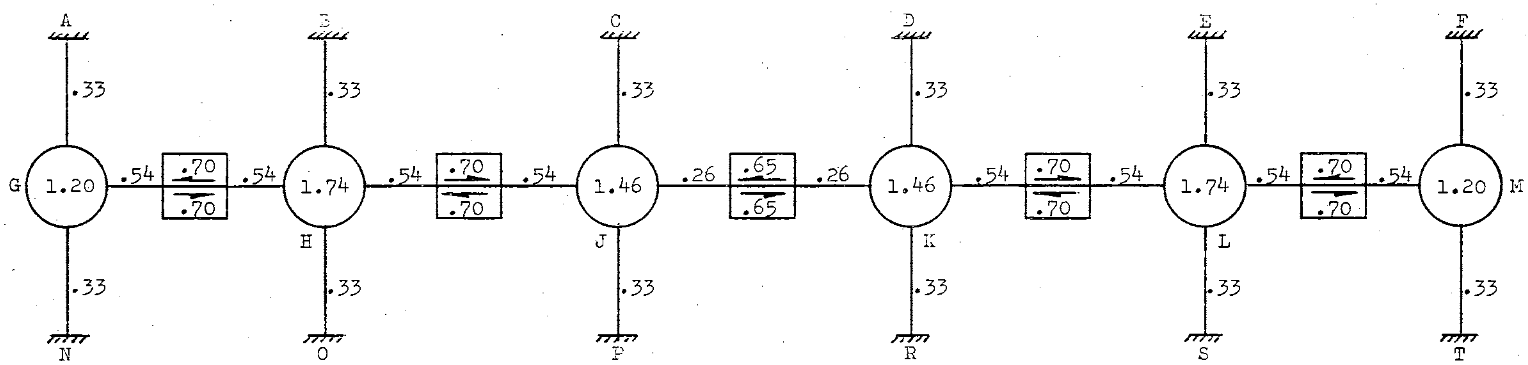
(c) Moment Distribution Procedure

A modified procedure is used here also in that adjacent joints are balanced alternately rather than simultaneously as in the regular moment distribution method. Member MU, having zero stiffness, is not shown in the table.

(d) Checking Procedure

A partial check is obtained if $\Sigma M_{ji} = 0$. However, any errors in carry-over values will, in general, go undetected.

EXAMPLE 1 MOMENT DISTRIBUTION SOLUTION TABLE 4



ji	GA	GN	GH	HG	HB	HO	HJ	JH	JC	JP	JK	KJ	KD	KR	KL	LK	LE	LS	LM	ML	MF	MT
D_{ji}	.275	.275	.450	.310	.190	.190	.310	.370	.226	.226	.178	.178	.226	.226	.370	.310	.190	.190	.310	.450	.275	.275
FM_{ji}	0.0	0.0	0.0	0.0	0.0	0.0	-41.4	+41.4	0.0	0.0	0.0	0.0	0.0	0.0	-41.4	+41.4	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	+ 9.0	+12.8	+ 7.9	+ 7.9	+12.8	+ 9.0	0.0	0.0	+ 4.8	+ 7.4	+ 9.3	+ 9.3	+15.3	+10.7	0.0	0.0	+ 5.7	+ 8.1	+ 5.0	+ 5.0
	- 2.5	- 2.5	- 4.0	- 2.8	0.0	0.0	-14.3	-20.4	-12.5	-12.5	- 9.8	- 6.4	0.0	0.0	-12.5	-17.9	-11.0	-11.0	-17.9	-12.5	0.0	0.0
	0.0	0.0	+ 3.7	+ 5.3	+ 3.3	+ 3.3	+ 5.3	+ 3.7	0.0	0.0	+ 2.2	+ 3.4	+ 4.3	+ 4.3	+ 7.0	+ 5.0	0.0	0.0	+ 3.9	+ 5.6	+ 3.4	+ 3.4
	- 1.0	- 1.0	- 1.7	- 1.2	0.0	0.0	- 1.5	- 2.2	- 1.3	- 1.3	- 1.1	- 0.7	0.0	0.0	- 2.0	- 2.8	- 1.7	- 1.7	- 2.8	- 2.0	0.0	0.0
	0.0	0.0	+ 0.6	+ 0.8	+ 0.5	+ 0.5	+ 0.8	+ 0.6	0.0	0.0	+ 0.3	+ 0.5	+ 0.6	+ 0.6	+ 1.0	+ 0.7	0.0	0.0	+ 0.6	+ 0.9	+ 0.5	+ 0.5
	- 0.1	- 0.1	- 0.3	0.0	0.0	0.0	0.0	- 0.3	- 0.2	- 0.2	- 0.2	0.0	0.0	0.0	0.0	- 0.4	- 0.2	- 0.2	- 0.4	0.0	0.0	0.0
M_{ji}	- 3.6	- 3.6	+ 7.3	+14.9	+11.7	+11.7	-38.3	+31.8	-14.0	-14.0	- 3.8	+ 4.2	+14.2	+14.2	-32.6	+36.7	-12.9	-12.9	-10.9	+ 0.1	+ 8.9	+ 8.9

PART V

EXAMPLE PROBLEM 2

The numerical procedure is now applied to a more complicated frame in order to clearly show the value of carry-over methods. A typical building frame (Fig. 7) is analyzed for the condition of symmetrical live load shown. This frame is similar to an interior bent of an office building which has been designed (5).

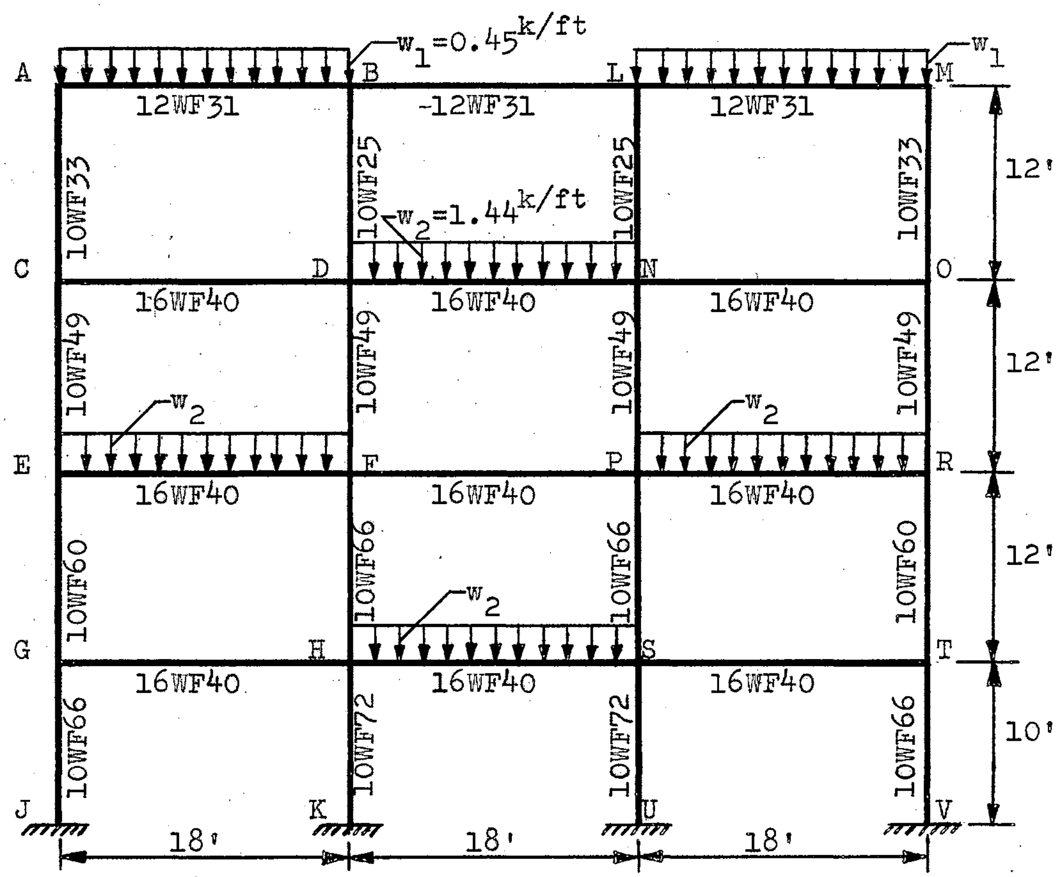


Figure 7

Multi-story Building Frame

Stiffnesses of the various members are shown in Table 5 with $E = 30(10)^6$. K_{ji} and ΣK_j (multiplied by E) are shown in Figure 8.

EXAMPLE 2	ABSOLUTE STIFFNESSES			TABLE 5
MEMBER	$I(in^4)$	L(in)	K/E (k-ft)	K"/E (k-ft)
10WF25	133.2	144	0.3083	–
10WF33	170.9	144	0.3956	–
10WF49	272.9	144	0.6317	–
10WF60	343.7	144	0.7956	–
10WF66	382.5	144	0.8854	–
10WF66	382.5	120	1.0625	–
10WF72	420.7	120	1.1686	–
12WF31	238.4	216	0.3679	0.1840
16WF40	515.5	216	0.7955	0.3978

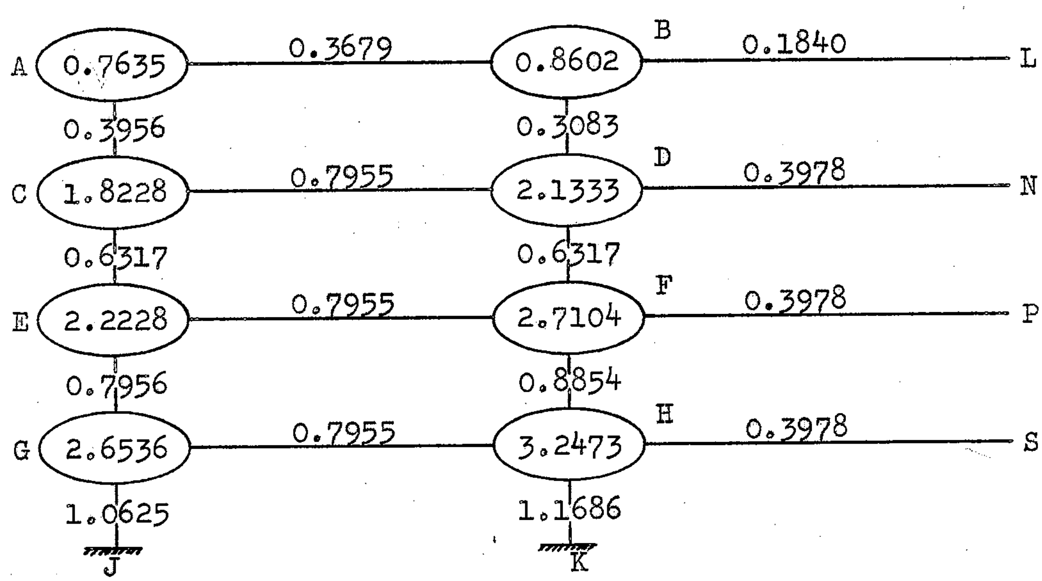


Figure 8

Stiffness Diagram

Distribution and carry-over factors, computed by referring to Figure 8, are shown below, with $C_{ji} = C_{ij} = 0.500$, except for C_{BL}, C_{DN}, C_{FP}, and C_{HS} which are equal to zero since the structure has been modified, taking advantage of the symmetry of structure and load.

EXAMPLE 2 DISTRIBUTION AND CARRY-OVER FACTORS TABLE 6

ji	D_{ji}	$r_{ji}=t_{ij}$	ji	D_{ji}	$r_{ji}=t_{ij}$
AB	.482	-.241	EF	.358	-.179
AC	.518	-.259	EG	.358	-.179
BA	.428	-.214	FD	.233	-.117
BD	.358	-.179	FE	.293	-.147
BL	.214	.000	FH	.327	-.163
CA	.218	-.109	FP	.147	.000
CD	.436	-.218	GE	.300	-.150
CE	.346	-.173	GH	.300	-.150
DB	.144	-.072	GJ	.400	-.200
DC	.373	-.186	HF	.273	-.136
DF	.296	-.148	HG	.245	-.122
DN	.186	.000	HK	.360	-.180
EC	.284	-.142	HS	.122	.000

1. Carry-over Slope Solution (Table 7)

Starting slopes are computed as follows:

$$E\theta_A^* = -\frac{-0.45(18)^2}{12(0.7635)} = +15.92$$

$$E\theta_B^* = -\frac{+0.45(18)^2}{12(0.8602)} = -14.13$$

$$E\theta_C^* = E\theta_G^* = 0$$

$$E\theta_D^* = -\frac{-1.44(18)^2}{12(2.1333)} = +18.23$$

$$E\theta_E^* = -\frac{-1.44(18)^2}{12(2.2228)} = +17.50$$

$$E\theta_F^* = -\frac{+1.44(18)^2}{12(2.7104)} = -14.35$$

$$E\theta_H^* = -\frac{-1.44(18)^2}{12(3.2473)} = +11.98$$

The modified procedure is again used in Table 7 and the check is established by $\Sigma M_j \doteq 0$.

2. Carry-over Moment Solution (Table 8)

Starting Moments are computed as follows:

$$m_A = -\frac{-0.45(18)^2}{12} = +12.15 \text{ k-ft} = -m_B$$

$$m_D = -\frac{-1.44(18)^2}{12} = +38.88 \text{ k-ft} = m_E = m_H = -m_F$$

The modified procedure is used as before and the check is established if $\Sigma M_{ji} \doteq 0$.

3. Moment Distribution Solution (Table 9)

Fixed-end Moments are:

$$FM_{AB} = -\frac{0.45(18)^2}{12} = -12.15 \text{ k-ft} = -FM_{BA}$$

$$FM_{DN} = -\frac{1.44(18)^2}{12} = -38.88 \text{ k-ft} = FM_{EF} = FM_{HS}$$

$$FM_{FE} = +\frac{1.44(18)^2}{12} = +38.88 \text{ k-ft}$$

The alternate method is also used here and the partial check made if $\Sigma M_{ji} \doteq 0$.

EXAMPLE 2 CARRY-OVER SLOPE SOLUTION TABLE 7

j	B		D			F			H	
t_{ji}	-.241	-.072	-.179	-.218	-.117	-.148	-.179	-.136	-.163	-.150
θ_j^*	-14.13			+18.23			-14.35		+11.98	
	- 3.41	- 3.26	+ 1.50	+ 1.63	+ 3.11	- 2.13	- 2.57	- 1.95	+ 2.86	+ 0.54
	- 1.56	- 1.12	+ 0.19	+ 0.59	+ 0.32	- 0.73	- 0.85	- 0.55	+ 0.29	+ 0.17
	- 0.31	- 0.20	+ 0.04	+ 0.11	+ 0.05	- 0.13	- 0.16	- 0.07	+ 0.05	+ 0.03
	- 0.06	- 0.04	+ 0.01	+ 0.02	+ 0.01	- 0.02	- 0.03	- 0.01	+ 0.01	+ 0.00
	- 0.01	- 0.01								
θ_j	-24.11			+25.81			-23.55		+15.93	

j	A		C			E			G	
t_{ji}	-.214	-.109	-.259	-.186	-.142	-.173	-.147	-.150	-.179	-.122
θ_j^*	+15.92			0.00			+17.50		0.00	
	+ 5.01	+ 2.27	- 1.74	- 3.97	- 3.03	+ 1.24	+ 3.76	+ 0.79	- 2.63	- 1.80
	+ 0.65	+ 0.82	- 0.79	- 1.36	- 1.00	+ 0.45	+ 0.38	+ 0.25	- 0.51	- 0.87
	+ 0.12	+ 0.15	- 0.16	- 0.24	- 0.19	+ 0.08	+ 0.06	+ 0.04	- 0.07	- 0.16
	+ 0.02	+ 0.03	- 0.03	- 0.04	- 0.03	+ 0.01	+ 0.01	+ 0.01	- 0.01	- 0.03
			- 0.01	- 0.01	- 0.01					
θ_j	+24.99			-12.61			+24.58		- 6.08	

ji	BA	BL	BD	DB	DC	DN	DF	FD	FE	FP	FH	HF	HS
K_{ji}	.368	.184	.308	.308	.796	.398	.632	.632	.796	.398	.885	.885	.398
$K_{ji}\theta_j$	- 8.87	- 4.44	- 7.43	+ 7.95	+20.52	+10.27	+16.30	-14.88	-18.75	- 9.37	-20.85	+14.11	+ 6.33
$C_{ij}K_{ij}\theta_i$	+ 4.60	0.00	+ 3.98	- 3.71	- 5.01	0.00	- 7.44	+ 8.15	+ 9.78	0.00	+ 7.05	-10.43	0.00
FM_{ji}	+12.15	0.00	0.00	0.00	0.00	-38.88	0.00	0.00	+38.88	0.00	0.00	0.00	-38.88
M_{ji}	+ 7.88	- 4.44	- 3.45	+ 4.24	+15.51	-28.61	+ 8.86	- 6.73	+29.91	- 9.37	-13.80	+ 3.68	-32.55
ji	AB	AC	CA	CD	CE	EC	EF	EG	GE	GJ	GH	HG	HK
K_{ji}	.368	.396	.396	.796	.632	.632	.796	.796	.796	1.062	.796	.796	1.169
$K_{ji}\theta_j$	+ 9.19	+ 9.89	- 4.99	-10.03	- 7.97	+15.54	+19.57	+19.57	- 4.84	- 6.46	- 4.84	+12.68	+18.62
$C_{ij}K_{ij}\theta_i$	- 4.44	- 2.49	+ 4.94	+10.26	+ 7.77	- 3.98	- 9.38	- 2.42	+ 9.78	0.00	+ 6.34	- 2.42	0.00
FM_{ji}	-12.15	0.00	0.00	0.00	0.00	0.00	-38.88	0.00	0.00	0.00	0.00	0.00	0.00
M_{ji}	- 7.40	+ 7.40	- 0.05	+ 0.23	- 0.20	+11.56	-28.69	+17.15	+ 4.94	- 6.46	+ 1.50	+10.26	+18.62

EXAMPLE 2 CARRY-OVER MOMENT SOLUTION TABLE 8

j	B		D			F			H	
r_{ji}	-.214	-.179	-.072	-.186	-.148	-.117	-.147	-.163	-.136	-.122
m_j	-12.15			+38.88			-38.88		+38.88	
	- 2.93	- 2.80	+ 3.20	+ 3.47	+ 6.66	- 5.75	- 6.96	- 5.29	+ 9.28	+ 1.76
	- 1.34	- 0.96	+ 0.41	+ 1.25	+ 0.68	- 1.97	- 2.31	- 1.50	+ 0.94	+ 0.55
	- 0.27	- 0.17	+ 0.08	+ 0.23	+ 0.11	- 0.35	- 0.43	- 0.20	+ 0.16	+ 0.09
	- 0.05	- 0.03	+ 0.01	+ 0.04	+ 0.02	- 0.06	- 0.08	- 0.03	+ 0.03	+ 0.02
	- 0.01	- 0.01				- 0.01	- 0.01	- 0.01		
JM_j	-20.72			+55.04			-63.84		+51.70	

j	A		C			E			G	
r_{ji}	-.241	-.259	-.109	-.218	-.173	-.142	-.179	-.179	-.150	-.150
m_j	+12.15			0.00			+38.88		0.00	
	+ 3.83	+ 1.73	- 3.15	- 7.23	- 5.53	+ 2.75	+ 8.37	+ 1.76	- 6.96	- 4.75
	+ 0.49	+ 0.63	- 1.44	- 2.48	- 1.83	+ 1.00	+ 0.85	+ 0.55	- 2.31	- 1.35
	+ 0.09	+ 0.12	- 0.29	- 0.44	- 0.34	+ 0.19	+ 0.14	+ 0.09	- 0.43	- 0.18
	+ 0.02	+ 0.02	- 0.05	- 0.08	- 0.06	+ 0.03	+ 0.02	+ 0.02	- 0.08	- 0.03
			- 0.01	- 0.01	- 0.01				- 0.01	- 0.01
JM_j	+19.08			-22.95			+54.65		-16.11	

ji	BA	BL	BD	DB	DC	DN	DF	FD	FE	FP	FH	HF	HS
D_{ji}	.428	.214	.358	.145	.373	.186	.296	.233	.293	.147	.327	.273	.122
$D_{ji}JM_j$	- 8.86	- 4.43	- 7.42	+ 7.99	+20.53	+10.25	+16.31	-14.87	-18.71	- 9.39	-20.87	+14.12	+ 6.32
$-r_{ij}JM_i$	+ 4.60	0.00	+ 3.96	- 3.71	- 5.01	0.00	- 7.48	+ 8.15	+ 9.78	0.00	+ 7.03	-10.43	0.00
FM_{ji}	+12.15	0.00	0.00	0.00	0.00	-38.88	0.00	0.00	+38.88	0.00	0.00	0.00	-38.88
M_{ji}	+ 7.89	- 4.43	- 3.46	+ 4.28	+15.52	-28.63	+ 8.83	- 6.72	+29.95	- 9.39	-13.84	+ 3.69	-32.56
ji	AB	AC	CA	CD	CE	EC	EF	EG	GE	GJ	GH	HG	HK
D_{ji}	.482	.518	.218	.436	.346	.284	.358	.358	.300	.400	.300	.245	.360
$D_{ji}JM_j$	+ 9.20	+ 9.88	- 5.01	-10.01	- 7.95	+15.53	+19.57	+19.57	- 4.83	- 6.44	- 4.83	+12.67	+18.62
$-r_{ij}JM_i$	- 4.43	- 2.50	+ 4.95	+10.25	+ 7.77	- 3.97	- 9.40	- 2.42	+ 9.78	0.00	+ 6.32	- 2.42	0.00
FM_{ji}	-12.15	0.00	0.00	0.00	0.00	0.00	-38.88	0.00	0.00	0.00	0.00	0.00	0.00
M_{ji}	- 7.38	+ 7.38	- 0.06	+ 0.24	- 0.18	+11.56	-28.71	+17.15	+ 4.95	- 6.44	+ 1.49	+10.25	+18.62

EXAMPLE 2 MOMENT DISTRIBUTION SOLUTION TABLE 9

CA	AC	AB	BA	BD	DB	BL
.218	.518	.482	.428	.358	.145	.214
		-12.15	+12.15	+ 2.82	+ 5.64	
+ 3.15	+ 6.30	+ 5.85	+ 2.93			
- 3.47	- 1.73	- 3.83	- 7.66	- 6.41	- 3.20	- 3.83
+ 1.44	+ 2.88	+ 2.68	+ 1.34	+ 0.96	+ 1.93	
- 1.25	- 0.63	- 0.49	- 0.99	- 0.82	- 0.41	- 0.49
+ 0.29	+ 0.58	+ 0.54	+ 0.27	+ 0.17	+ 0.34	
- 0.23	- 0.12	- 0.09	- 0.19	- 0.16	- 0.08	- 0.09
+ 0.05	+ 0.11	+ 0.10	+ 0.05	+ 0.03	+ 0.06	
- 0.04			- 0.03	- 0.03		- 0.02
- 0.06	+ 7.39	- 7.39	+ 7.87	- 3.44	+ 4.28	- 4.43

EC	CE	CD	DC	DF	FD	DN
.284	.346	.436	.373	.296	.233	.186
+11.04	+ 5.52	+ 7.25	+14.50	+11.51	+ 5.75	-38.88
- 2.76	- 5.51	- 6.94	- 3.47	- 6.63	-13.26	+ 7.23
+ 3.65	+ 1.83	+ 2.48	+ 4.96	+ 3.94	+ 1.97	+ 2.47
- 0.98	- 1.99	- 2.51	- 1.25	- 0.67	- 1.35	
+ 0.68	+ 0.34	+ 0.43	+ 0.87	+ 0.69	+ 0.34	+ 0.43
- 0.18	- 0.37	- 0.46	- 0.23	- 0.11	- 0.23	
+ 0.11	+ 0.06	+ 0.08	+ 0.16	+ 0.12	+ 0.06	+ 0.08
	- 0.07	- 0.08			- 0.04	
+11.56	- 0.19	+ 0.25	+15.55	+ 8.85	- 6.76	-28.67

GE	EG	EF	FE	FH	HF	FP
.300	.358	.358	.293	.327	.273	.147
		-38.88	+38.88	+ 5.31	+10.62	
+ 6.96	+13.92	+13.92	+ 6.96			
- 3.52	- 1.76	- 8.34	-16.67	-18.60	- 9.30	- 8.37
+ 2.30	+ 4.61	+ 4.61	+ 2.30	+ 1.51	+ 3.02	
- 1.10	- 0.55	- 0.85	- 1.69	- 1.89	- 0.95	- 0.85
+ 0.43	+ 0.85	+ 0.85	+ 0.43	+ 0.20	+ 0.41	
- 0.18	- 0.09	- 0.14	- 0.28	- 0.32	- 0.16	- 0.14
+ 0.07	+ 0.15	+ 0.15	+ 0.07	+ 0.03	+ 0.07	
- 0.03			- 0.05	- 0.05		- 0.02
+ 4.93	+17.13	-28.68	+29.95	-13.81	+ 3.71	- 9.38

GJ	GH	HG	HK	HS
.400	.300	.245	.360	.122
	+ 4.76	+ 9.52	+14.00	-38.88
- 4.69	- 3.52	- 1.76		+ 4.75
	+ 1.36	+ 2.71	+ 3.98	+ 1.35
- 1.46	- 1.10	- 0.55		+ 0.18
	+ 0.18	+ 0.37	+ 0.54	
- 0.25	- 0.18	- 0.09		
	+ 0.03	+ 0.06	+ 0.09	+ 0.03
- 0.04	- 0.03			
- 6.44	+ 1.50	+10.26	+18.61	-32.57

PART VI

SUMMARY AND CONCLUSIONS

This study is the initial extension to continuous frames of the carry-over theory developed by Professor Jan J. Tuma for the analysis of continuous beams (2).

In this case the redundants are taken to be the unknown joint rotations (or joint moments) and are computed by the carry-over process to a desired degree of accuracy. The physical interpretation of the process follows easily and the numerical results are self-checking and easy to tabulate. Due to the simplicity of the process, errors are easily detected.

Comparison of the computations in Examples 1 and 2 show the two carry-over methods and the moment distribution method to have the same rates of convergency. Another conclusion is that for the computation of end moments both carry-over methods are similarily accurate and easy to apply. Although the carry-over slope method does not necessarily require the computation of distribution factors, the ability to work with moments rather than slopes may off-set this advantage and the difference between the two methods becomes negligible.

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